

Corso di laurea in Geologia
Istituzioni di matematiche
Esercizi n. 1617/2/7

Regole di integrazione:

$$\begin{aligned} \int (f(x) + g(x)) dx &= \int f(x) dx + \int g(x) dx, \\ \int a \cdot f(x) dx &= a \cdot \int f(x) dx, & \int x^n dx &= \frac{1}{n+1} x^{n+1} + C, \\ \int \sin(x) dx &= -\cos(x) + C, & \int \cos(x) dx &= \sin(x) + C, \\ \int \frac{1}{\cos^2(x)} dx &= \tan(x) + C, & \int \frac{1}{x} dx &= \log(|x|) + C, \\ \int e^x dx &= e^x + C, & \int \log(x) dx &= x \log(x) - x + C, \\ \int x^a dx &= \frac{1}{a+1} x^{a+1} + C, \quad a \neq -1, & \int \sqrt{x} dx &= \frac{2}{3} x \sqrt{x} + C, \\ \int \frac{1}{\sqrt{1-x^2}} dx &= \arcsin(x) + C, & \int \frac{1}{1+x^2} dx &= \arctan(x) + C, \\ \int \frac{f'(x)}{f(x)} dx &= \log(|f(x)|) + C, & \int f(x)^n \cdot f'(x) dx &= \frac{1}{n+1} f(x)^{n+1} + C \end{aligned}$$

Formula di integrazione per parti:

$$\int f'(x) \cdot g(x) dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x) dx$$

$$\int f(g(x)) \cdot g'(x) dx = F(g(x)) + c \quad (\text{dove } F(x) \text{ è una primitiva di } f(x)).$$

1. Integrali indefiniti (immediati dalle formule):

$$\int 8x^3 + 6x, \quad \int (\sin(x) + 2 \cos(x)) dx \quad \int \frac{1}{x^2} dx, \quad \int \frac{4}{x^3} dx.$$

Risp.:

$$2x^4 + 3x^2 + c, \quad -\cos(x) + 2 \sin(x) + c, \quad -\frac{1}{x} + c, \quad -\frac{2}{x^2} + c.$$

2. Integrali indefiniti (immediati dalle formule):

$$\int \frac{3}{\cos^2(x)} dx, \quad \int \sqrt{x^3} dx, \quad \int \frac{1}{\sqrt{1-x^2}} dx,$$

Risp.:

$$3 \tan(x) + c, \quad \frac{2}{5} x^2 \sqrt{x} + c, \quad \arcsin(x) + c$$

3. Formula $\int (f'(x)/f(x)) dx = \log(|f(x)|) + c$:

$$\int \frac{3x}{x^2 + 8} dx \quad \int \frac{\sin(x)}{\cos(x) + 2} dx \quad \int \frac{1 + \sin(x)}{x - \cos(x)} dx.$$

Risp.:

$$\frac{3}{2} \log(x^2 + 8) + c, \quad -\log(\cos(x) + 2) + c, \quad \log|x - \cos(x)| + c$$

4. Integrazione per parti:

$$\int 2x \sin(x) dx, \quad \int x \log(x) dx, \quad \int x^2 e^x dx.$$

Risp.:

$$-2x \cos(x) + 2 \sin(x) + c, \quad \frac{1}{2} x^2 \log(x) - \frac{1}{4} x^2 + c, \quad (x^2 - 2x + 2)e^x + c.$$

5. Formula $\int f(g(x)) \cdot g'(x) dx = F(g(x)) + c$:

$$\int \sin(4x) dx, \quad \int x^2 \cos(x^3) dx, \quad \int x \log(x^2 + 2) dx \\ \int \frac{2x}{1+x^4} dx, \quad \int \frac{2x}{\sqrt{1-x^4}} dx \quad \int x \sqrt{x^2 + 2} dx.$$

Risp.:

$$-\frac{1}{4} \cos(4x) + c, \quad \frac{1}{3} \sin(x^3) + c, \quad -\frac{1}{2} x^2 + \frac{1}{2} (x^2 + 2) \log(x^2 + 2) - 1 + c \\ \arctan(x^2) + c, \quad \arcsin(x^2) + c, \quad \frac{1}{3} \sqrt{(x^2 + 2)^3} + c.$$

6. Miscellanea:

$$\int \frac{x+3}{x+5} dx, \quad \int (x+2) \sin(3x) dx, \quad \int (x-1)e^{2x+3} dx$$

Risp.:

$$x-2 \log(x+5)+c, \quad -\frac{1}{3}x \cos(3x)-\frac{2}{3} \cos(3x)+\frac{1}{9} \sin(3x)+c, \quad \frac{1}{4}(2x-3)e^{(2x+3)}+c.$$