VETTORI GEOMETRICI (FORZE TRIDIMENSIONALI)

Somma di due vettori; la seguente procedura "disegna" la somma di due vettori definiti dal loro secondo estremo; rispettivamente P e Q.

```
> with(plots):
> somma:=proc(P,Q)
  local OO,u,v,upiuv,uu,vv,M;
  OO:=[0,0,0];
  u:=[OO,P];
  uu:=[Q,P+Q];
  v:=[P,P+Q];
  vv:=[OO,Q];
  upiuv:=[OO,P+Q];
  M:=PLOT3D(CURVES(u,v,COLOR(RGB,0,1,0)),CURVES(upiuv,COLOR(RGB,1,0,
  0)),CURVES(uu,vv,COLOR(RGB,0,0,0)),AXESSTYLE(FRAME),AXESLABELS("x"
  ,"y","z"),POINTS(P,P+Q,Q,SYMBOL(BOX),COLOR(RGB,0,0,1)),TEXT(P-[1,1
  ,1],"u"),TEXT(P+[0,1,2],"v"),TEXT((P+Q)-[2,2,1.5],"u+v"),COLOR(ZHU
  E)):
  display3d([M],orientation=[-52,49]);
  end;
```

somma := **proc**(*P*, *Q*)

local *OO*, *u*, *v*, *upiuv*, *uu*, *vv*, *M*;

$OO := [0, 0, 0]$;

$u := [OO, P]$;

$uu := [Q, P+Q]$;

$v := [P, P+Q]$;

$vv := [OO, Q]$;

$upiuv := [OO, P+Q]$;

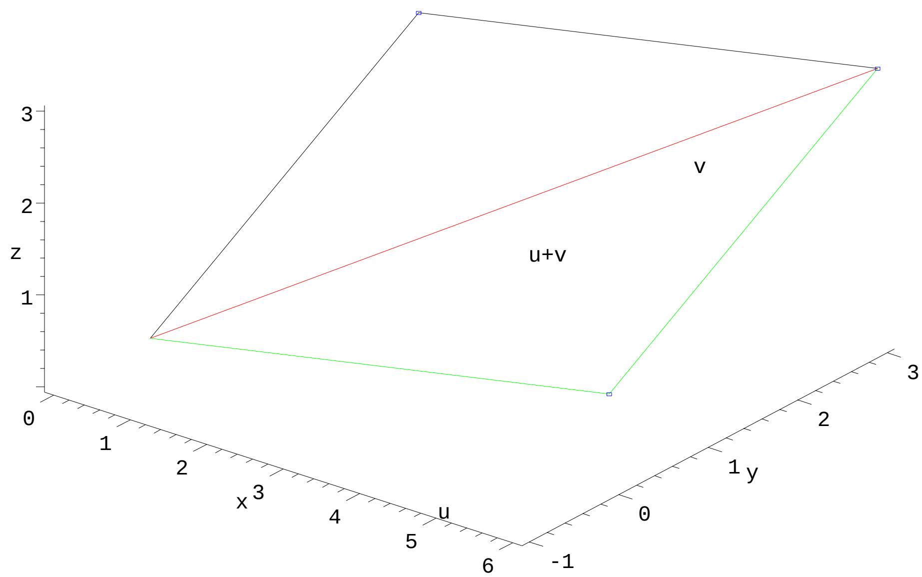
M := PLOT3D(CURVES(*u*, *v*, COLOR(*RGB*, 0, 1, 0)),
CURVES(*upiuv*, COLOR(*RGB*, 1, 0, 0)), CURVES(*uu*, *vv*, COLOR(*RGB*, 0, 0, 0)),
AXESSTYLE(*FRAME*), AXESLABELS("x", "y", "z"),
POINTS(*P*, *P* + *Q*, *Q*, SYMBOL(*BOX*), COLOR(*RGB*, 0, 0, 1)),
TEXT(*P* + [-1, -1, -1], "u"), TEXT(*P* + [0, 1, 2], "v"),
TEXT(*P* + *Q* + [-2, -2, -1.5], "u+v"), COLOR(*ZHUE*));

display3d([*M*], *orientation* = [-52, 49])

end

Esempio: u:=[6,0,4], v:=[0,5,5] sono in verde; u+v:=[6,5,9] in rosso.

```
> somma([6,0,1],[0,3,2]);
```



Proprieta' associativa della somma. Verifica grafica.

```
> prop_ass:=proc(P,Q,R)
  local OO,u,v,w,upiuv,upiuvpiuw,vpiuw,M;
  OO:=[0,0,0];
  u:=[OO,P];
  v:=[P,P+Q];
  w:=[P+Q,P+Q+R];
  upiuv:=[OO,P+Q];
  upiuvpiuw:=[OO,P+Q+R];
  vpiuw:=[P,P+Q+R];
  M:=PLOT3D(CURVES(u,v,w,COLOR(RGB,0,1,0)),CURVES(upiuv,vpiuw,COLOR(
  RGB,0,0,1)),CURVES(upiuvpiuw,COLOR(RGB,1,0,0)),AXESSTYLE(FRAME),AX
  ESLABELS("x","y","z"),POINTS(P,P+Q,P+Q+R,SYMBOL(BOX),COLOR(RGB,0,0
  ,0)),TEXT(P-[1,1,1],"u"),TEXT(P+[0,1,2],"v"),TEXT((P+Q)+[1,1,1],"w
  "),TEXT((P+Q)-[2,2,1.5],"u+v"),TEXT((P+Q+R)-[0,5,1.5],"v+w"),TEXT(
  P-[1,1,1],"u"),COLOR(ZHUE)):
  display3d([M],orientation=[-52,49]);
  end;
```

prop_ass := **proc**(*P*, *Q*, *R*)

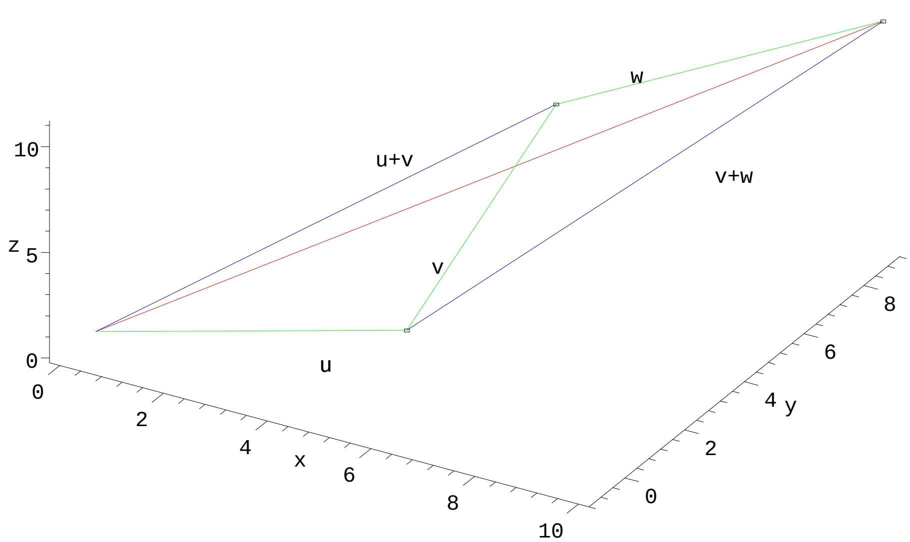
local *OO*, *u*, *v*, *w*, *upiuv*, *upiuvpiuw*, *vpiuw*, *M*;

OO := [0, 0, 0];

```
u := [OO, P];
v := [P, P + Q];
w := [P + Q, P + Q + R];
upiuv := [OO, P + Q];
upiuvpiuw := [OO, P + Q + R];
vpiuw := [P, P + Q + R];
M := PLOT3D(CURVES(u, v, w, COLOR(RGB, 0, 1, 0)),
    CURVES(upiuv, vpiuw, COLOR(RGB, 0, 0, 1)),
    CURVES(upiuvpiuw, COLOR(RGB, 1, 0, 0)), AXESSTYLE(FRAME),
    AXESLABELS("x", "y", "z"),
    POINTS(P, P + Q, P + Q + R, SYMBOL(BOX), COLOR(RGB, 0, 0, 0)),
    TEXT(P + [-1, -1, -1], "u"), TEXT(P + [0, 1, 2], "v"), TEXT(P + Q + [1, 1, 1], "w"),
    TEXT(P + Q + [-2, -2, -1.5], "u+v"), TEXT(P + Q + R + [0, -5, -1.5], "v+w"),
    TEXT(P + [-1, -1, -1], "u"), COLOR(ZHUE));
display3d([M], orientation = [-52, 49])
end
```

Esempio: Ai due vettori di prima aggiungiamo il vettore w:=[4,4,2];

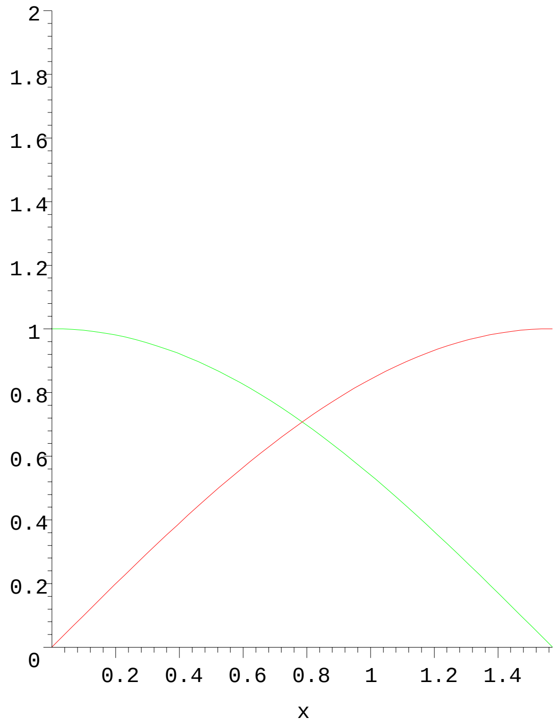
```
> prop_ass([6,0,4],[0,5,5],[4,4,2]);
```



SPAZIO VETTORIALE DELLE FUNZIONI DEFINITE SU UN INTERVALLO

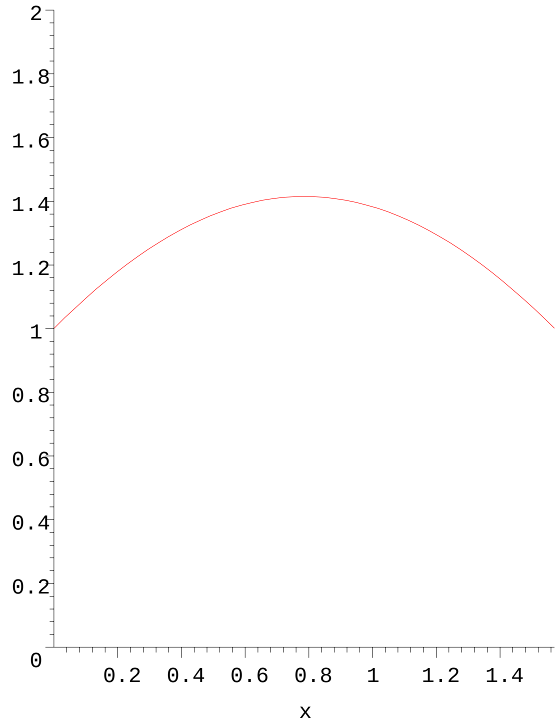
Prendiamo sin(x) (grafico rosso) e cos(x) (grafico verde) definite sull'intervallo [0, Pi/2].

```
> plot([sin(x),cos(x)],x=0..Pi/2,0..2,scaling=CONSTRAINED);
```



Sommiamo sin+cos; otteniamo la funzione seguente:

```
> plot(sin(x)+cos(x),x=0..Pi/2,0..2,scaling=CONSTRAINED);
```



Verifica "sperimentale" della proprieta' associativa. Prendiamo h(x):=x; calcoliamo (sin+cos)+h:

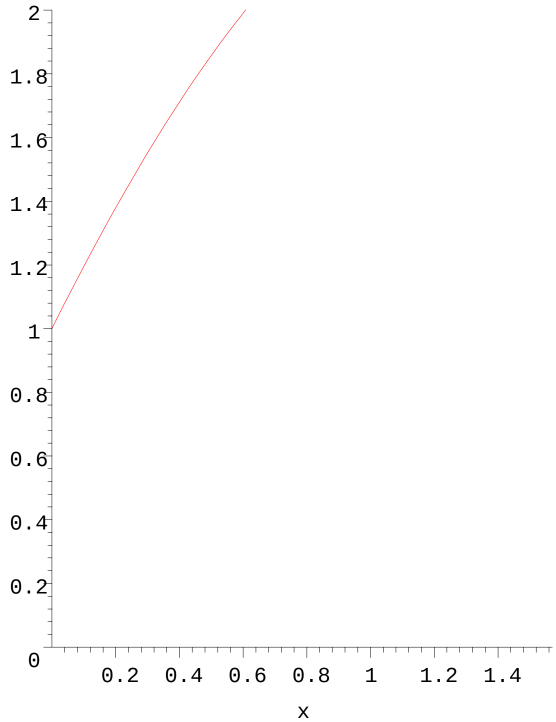
```
> h(x):=x;
```

$$h(x) := x$$

```
> f(x):=sin(x)+cos(x);
```

$$f(x) := \sin(x) + \cos(x)$$

```
> plot(f(x)+h(x),x=0..Pi/2,0..2,scaling=CONSTRAINED);
```

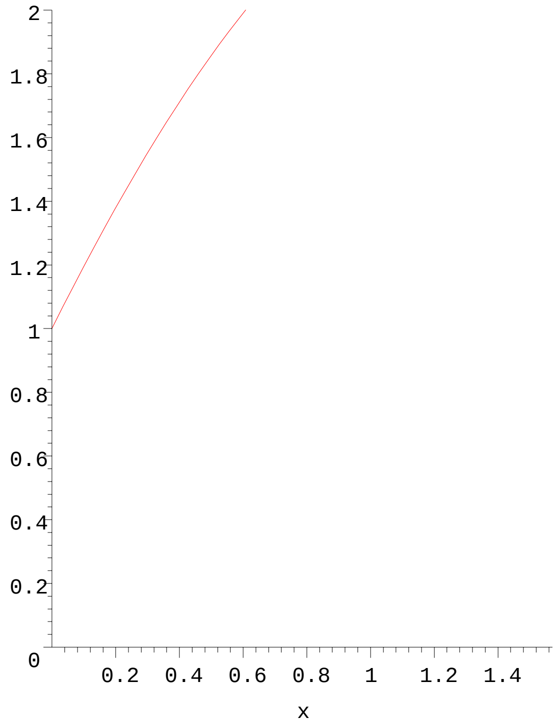


Calcoliamo ora sin+(cos+h);

```
> g(x):=cos(x)+h(x);
```

$$g(x) := \cos(x) + x$$

```
> plot(sin(x)+g(x),x=0..Pi/2,0..2,scaling=CONSTRAINED);
```

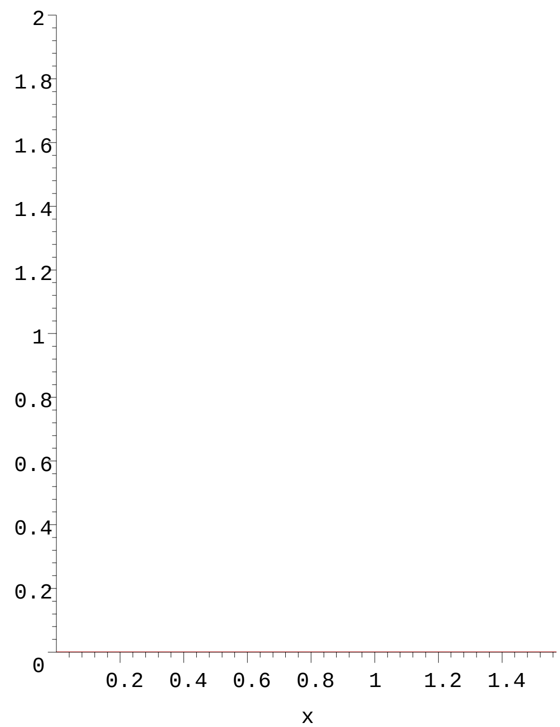


Funzione nulla: OO(x):=0;

```
> OO(x):=0;
```

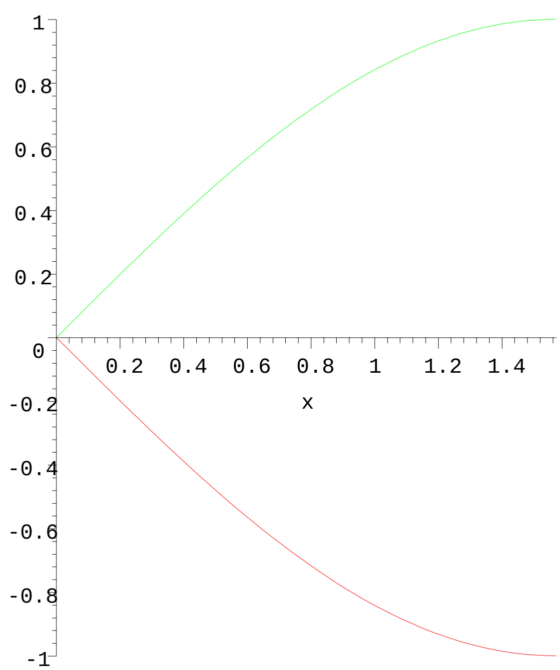
$$OO(x) := 0$$

```
> plot(OO(x),x=0..Pi/2,0..2,scaling=CONSTRAINED);
```



Funzione opposta. Se f(x):=sin(x), allora:

```
> plot([-sin(x),sin(x)],x=0..Pi/2,-1..1,scaling=CONSTRAINED);
```



SPAZIO VETTORIALE DELLE MATRICI

Manipolazione simbolica delle matrici. Caso di matrici 3x4; esempio:

```
> with(linalg):
> A:=matrix([[a[1,1],a[1,2],a[1,3],a[1,4]],[a[2,1],a[2,2],a[2,3],a[2,4]],[a[3,1],a[3,2],a[3,3],a[3,4]]]);
```

$$A := \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \end{bmatrix}$$

```
> B:=matrix([[b[1,1],b[1,2],b[1,3],b[1,4]],[b[2,1],b[2,2],b[2,3],b[2,4]],[b[3,1],b[3,2],b[3,3],b[3,4]]]);
```

$$B := \begin{bmatrix} b_{1,1} & b_{1,2} & b_{1,3} & b_{1,4} \\ b_{2,1} & b_{2,2} & b_{2,3} & b_{2,4} \\ b_{3,1} & b_{3,2} & b_{3,3} & b_{3,4} \end{bmatrix}$$

```
> C:=matrix([[c[1,1],c[1,2],c[1,3],c[1,4]],[c[2,1],c[2,2],c[2,3],c[2,4]],[c[3,1],c[3,2],c[3,3],c[3,4]]]);
```

$$C := \begin{bmatrix} c_{1,1} & c_{1,2} & c_{1,3} & c_{1,4} \\ c_{2,1} & c_{2,2} & c_{2,3} & c_{2,4} \\ c_{3,1} & c_{3,2} & c_{3,3} & c_{3,4} \end{bmatrix}$$

```
> H:=matadd(A,B);
```

$$H := \begin{bmatrix} a_{1,1}+b_{1,1} & a_{1,2}+b_{1,2} & a_{1,3}+b_{1,3} & a_{1,4}+b_{1,4} \\ a_{2,1}+b_{2,1} & a_{2,2}+b_{2,2} & a_{2,3}+b_{2,3} & a_{2,4}+b_{2,4} \\ a_{3,1}+b_{3,1} & a_{3,2}+b_{3,2} & a_{3,3}+b_{3,3} & a_{3,4}+b_{3,4} \end{bmatrix}$$

(A+B)+C:

```
> DD:=matadd(H,C);
```

$$DD := \begin{bmatrix} a_{1,1}+b_{1,1}+c_{1,1} & a_{1,2}+b_{1,2}+c_{1,2} & a_{1,3}+b_{1,3}+c_{1,3} & a_{1,4}+b_{1,4}+c_{1,4} \\ a_{2,1}+b_{2,1}+c_{2,1} & a_{2,2}+b_{2,2}+c_{2,2} & a_{2,3}+b_{2,3}+c_{2,3} & a_{2,4}+b_{2,4}+c_{2,4} \\ a_{3,1}+b_{3,1}+c_{3,1} & a_{3,2}+b_{3,2}+c_{3,2} & a_{3,3}+b_{3,3}+c_{3,3} & a_{3,4}+b_{3,4}+c_{3,4} \end{bmatrix}$$

```
> K:=matadd(B,C);
```

$$K := \begin{bmatrix} b_{1,1}+c_{1,1} & b_{1,2}+c_{1,2} & b_{1,3}+c_{1,3} & b_{1,4}+c_{1,4} \\ b_{2,1}+c_{2,1} & b_{2,2}+c_{2,2} & b_{2,3}+c_{2,3} & b_{2,4}+c_{2,4} \\ b_{3,1}+c_{3,1} & b_{3,2}+c_{3,2} & b_{3,3}+c_{3,3} & b_{3,4}+c_{3,4} \end{bmatrix}$$

A+(B+C):

```
> matadd(A,K);
```

$$\begin{bmatrix} a_{1,1}+b_{1,1}+c_{1,1} & a_{1,2}+b_{1,2}+c_{1,2} & a_{1,3}+b_{1,3}+c_{1,3} & a_{1,4}+b_{1,4}+c_{1,4} \\ a_{2,1}+b_{2,1}+c_{2,1} & a_{2,2}+b_{2,2}+c_{2,2} & a_{2,3}+b_{2,3}+c_{2,3} & a_{2,4}+b_{2,4}+c_{2,4} \\ a_{3,1}+b_{3,1}+c_{3,1} & a_{3,2}+b_{3,2}+c_{3,2} & a_{3,3}+b_{3,3}+c_{3,3} & a_{3,4}+b_{3,4}+c_{3,4} \end{bmatrix}$$

Prodotto per uno scalare:

```
> scalarmul(A,c);
```

$$\begin{bmatrix} c\,a_{1,1} & c\,a_{1,2} & c\,a_{1,3} & c\,a_{1,4} \\ c\,a_{2,1} & c\,a_{2,2} & c\,a_{2,3} & c\,a_{2,4} \\ c\,a_{3,1} & c\,a_{3,2} & c\,a_{3,3} & c\,a_{3,4} \end{bmatrix}$$

Manipolazione numerica; caso delle matrici 4x5:

```
> A:=matrix([[1,2,3,2,4],[0,3,4,5,2],[2,1,1,0,2],[0,0,3,4,1]]);
```

$$A := \begin{bmatrix} 1 & 2 & 3 & 2 & 4 \\ 0 & 3 & 4 & 5 & 2 \\ 2 & 1 & 1 & 0 & 2 \\ 0 & 0 & 3 & 4 & 1 \end{bmatrix}$$

```
> B:=matrix([[1,2,1,2,2],[0,2,0,0,4],[2,1,1,1,3],[1,1,3,1,2]]);
```

$$B := \begin{bmatrix} 1 & 2 & 1 & 2 & 2 \\ 0 & 2 & 0 & 0 & 4 \\ 2 & 1 & 1 & 1 & 3 \\ 1 & 1 & 3 & 1 & 2 \end{bmatrix}$$

```
> matadd(A,B);
```

$$\begin{bmatrix} 2 & 4 & 4 & 4 & 6 \\ 0 & 5 & 4 & 5 & 6 \\ 4 & 2 & 2 & 1 & 5 \\ 1 & 1 & 6 & 5 & 3 \end{bmatrix}$$

```
> scalarmul(A,4);
```

$$\begin{bmatrix} 4 & 8 & 12 & 8 & 16 \\ 0 & 12 & 16 & 20 & 8 \\ 8 & 4 & 4 & 0 & 8 \\ 0 & 0 & 12 & 16 & 4 \end{bmatrix}$$

SPAZIO VETTORIALE R^n

Somma di n-uple:

```
> u:=[a,b,c,d,e];
```

$$u := [a, b, c, d, e]$$

```
> v:=[m,n,p,q,r];
```

$$v := [m, n, p, q, r]$$

```
> u+v;
```

$$[m + a, n + b, p + c, q + d, r + e]$$

Prodotto per uno scalare:

```
> expand(h*u);
```

$$[a\, h, b\, h, c\, h, d\, h, e\, h]$$

Effetto di h*u:

```
> h*u;
```

$$h\, [a, b, c, d, e]$$

SPAZIO VETTORIALE DEI POLINOMI R[x]

I polinomi (ad una indeterminata sul campo reale) sono scritture del tipo:

```
> f:=2*x^4+3*x^3+4*x+5;
```

$$f := 2\, x^4 + 3\, x^3 + 4\, x + 5$$

L'insieme dei polinomi a coefficienti reali si denota con R[x]. Due polinomi si possono sommare:

```
> g:=-2*x^4+x^3+x^2+4*x;
```

$$g := -2\, x^4 + x^3 + x^2 + 4\, x$$

```
> f+g;
```

$$4\, x^3 + 8\, x + 5 + x^2$$

Si puo' moltiplicare un polinomio per un numero:

```
> 3*f;
```

$$6\, x^4 + 9\, x^3 + 12\, x + 15$$

```
>
```