CENTRALI E CITTA'

Problema:

Ci sono 4 centrali elettriche C_1, C_2, C_3 e C_4 che producono, in un giorno, rispettivamente a_1, a_2, a_3, a_4 Kw.
L'energia prodotta dalle centrali viene inviata a 3 citta' T_1, T_2 e T_3 che consumano, rispettivamente b_1, b_2 e b_3 Kw giornalieri. Quanti Kw x_ij bisogna inviare da C_i a T_j in modo da esaurire tutta la produzione delle centrali e da soddisfare tutte le richieste di energia delle citta' ?

Nel seguito le 12 colonne di A sono relative alle variabili:
x_11, x_12, x_13, x_21, x_22, x_23, x_31, x_32, x_33, x_41, x_42, x_43.

```
> with(linalg):
> A:=matrix([[1,1,1,0,0,0,0,0,0,0,0,0],[0,0,0,1,1,1,0,0,0,0,0,0],[0,
  0,0,0,0,0,1,1,1,0,0,0],[0,0,0,0,0,0,0,0,0,1,1,1],[1,0,0,1,0,0,1,0,
  0,1,0,0],[0,1,0,0,1,0,0,1,0,0,1,0],[0,0,1,0,0,1,0,0,1,0,0,1]]);
```

$$A := \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

```
> rank(A);
```

$$6$$

Allora le sette righe di A sono linearmente dipendenti. Troviamo quelle indipendenti.

```
> B:=basis(A,'rowspace');
```

$$B := [[1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1], [1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0], [0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0]]$$

Sono le prime sei. Vediamo come la settima si scrive come combinazione lineare delle rimanenti.

```
> BB:=matrix(B);
```

$$BB := \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

```
> linsolve(transpose(BB),vector([0,0,1,0,0,1,0,0,1,0,0,1]));
```

$$[1, 1, 1, 1, -1, -1]$$

Cioe' la settima riga di A e' combinazione lineare delle rimanenti con i coefficienti scritti sopra.

La seguente procedura risolve il sistema lineare. a e' la lista delle produzioni delle centrali. b e' la lista dei consumi delle citta'.
Per avere soluzioni deve essere a_1 + a_2 + a_3 + a_4 - b_1 - b_2 = b_3.

```
> cent_cit:=proc(a,b)
  local A,u,v,c,d,l;
  A:=matrix([[1,1,1,0,0,0,0,0,0,0,0,0],[0,0,0,1,1,1,0,0,0,0,0,0],[0,
  0,0,0,0,0,1,1,1,0,0,0],[0,0,0,0,0,0,0,0,0,1,1,1],[1,0,0,1,0,0,1,0,
  0,1,0,0],[0,1,0,0,1,0,0,1,0,0,1,0],[0,0,1,0,0,1,0,0,1,0,0,1]]);
  u:=matrix([a]);
  v:=matrix([b]);
  c:=concat(u,v);
  d:=convert(c,vector);
  l:=linsolve(A,d);
  end;
```

cent_cit := **proc**(*a*, *b*)

local *A*, *u*, *v*, *c*, *d*, *l*;

A := matrix([[1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1],
[1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0], [0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0],
[0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1]]);

u := matrix([*a*]);

v := matrix([*b*]);

c := concat(*u*, *v*);

d := convert(*c*, *vector*);

l := linsolve(*A*, *d*)

end

Facciamo un esempio: Supponiamo che a_1:=300000; a_2:=200000; a_3:=600000; a_4:=400000; b_1:=20000; b_2:=70000; allora:

```
> b_3:=300000+200000+600000+400000-20000-70000;
```

$$b\_3 := 1410000$$

Dunque sara':

```
> C:=cent_cit([300000,200000,600000,400000],[20000,70000,1410000]);
```

$$C := [-1180000 + \_t_1 + \_t_3 + \_t_5 + \_t_2 + \_t_4 + \_t_6,\ 70000 - \_t_1 - \_t_3 - \_t_5,\ 1410000 - \_t_2 - \_t_4 - \_t_6,$$
$$200000 - \_t_1 - \_t_2,\ \_t_1,\ \_t_2,\ 600000 - \_t_3 - \_t_4,\ \_t_3,\ \_t_4,\ 400000 - \_t_5 - \_t_6,\ \_t_5,\ \_t_6]$$

```
> x_11:=C[1];
```

$$x\_11 := -1180000 + \_t_1 + \_t_3 + \_t_5 + \_t_2 + \_t_4 + \_t_6$$

```
> x_12:=C[2];
```

$$x\_12 := 70000 - \_t_1 - \_t_3 - \_t_5$$

```
> x_13:=C[3];
```

$$x\_13 := 1410000 - \_t_2 - \_t_4 - \_t_6$$

```
> x_21:=C[4];
```

$$x\_21 := 200000 - \_t_1 - \_t_2$$

```
> x_22:=C[5];
```

$$x\_22 := \_t_1$$

```
> x_23:=C[6];
```

$$x\_23 := \_t_2$$

```
> x_31:=C[7];
```

$$x\_31 := 600000 - \_t_3 - \_t_4$$

```
> x_32:=C[8];
```

$$x\_32 := \_t_3$$

```
> x_33:=C[9];
```

$$x\_33 := \_t_4$$

```
> x_41:=C[10];
```

$$x\_41 := 400000 - \_t_5 - \_t_6$$

```
> x_42:=C[11];
```

$$x\_42 := \_t_5$$

```
> x_43:=C[12];
```

$$x\_43 := \_t_6$$

Le possibili scelte dipendono da valori arbitrari dei parametri t_1,...t_6 tali che le soluzioni siano positive.
Prendiamoli, per esempio, tutti 1000. Allora:

```
> eval(x_11,[_t[1]=1000,_t[2]=100000,_t[3]=1000,_t[4]=100000,_t[5]=1
  000,_t[6]=100000]);
```

```
                                    -877000
> eval(x_12,[_t[1]=1000,_t[2]=100000,_t[3]=1000,_t[4]=100000,_t[5]=1
  000,_t[6]=100000]);
                                     67000
> eval(x_13,[_t[1]=1000,_t[2]=100000,_t[3]=1000,_t[4]=100000,_t[5]=1
  000,_t[6]=100000]);
                                    1110000
> eval(x_21,[_t[1]=1000,_t[2]=100000,_t[3]=1000,_t[4]=100000,_t[5]=1
  000,_t[6]=100000]);
                                     99000
> eval(x_22,[_t[1]=1000,_t[2]=100000,_t[3]=1000,_t[4]=100000,_t[5]=1
  000,_t[6]=100000]);
                                      1000
> eval(x_23,[_t[1]=1000,_t[2]=100000,_t[3]=1000,_t[4]=100000,_t[5]=1
  000,_t[6]=100000]);
                                     100000
> eval(x_31,[_t[1]=1000,_t[2]=100000,_t[3]=1000,_t[4]=100000,_t[5]=1
  000,_t[6]=100000]);
                                     499000
> eval(x_32,[_t[1]=1000,_t[2]=100000,_t[3]=1000,_t[4]=100000,_t[5]=1
  000,_t[6]=100000]);
                                      1000
> eval(x_33,[_t[1]=1000,_t[2]=100000,_t[3]=1000,_t[4]=100000,_t[5]=1
  000,_t[6]=100000]);
                                     100000
> eval(x_41,[_t[1]=1000,_t[2]=100000,_t[3]=1000,_t[4]=100000,_t[5]=1
  000,_t[6]=100000]);
                                     299000
> eval(x_42,[_t[1]=1000,_t[2]=100000,_t[3]=1000,_t[4]=100000,_t[5]=1
  000,_t[6]=100000]);
                                      1000
> eval(x_43,[_t[1]=1000,_t[2]=100000,_t[3]=1000,_t[4]=100000,_t[5]=1
  000,_t[6]=100000]);
                                     100000
```

In questo caso, i risultati si possono interpretare cosi':

C_1 non invia corrente a T_1, invia 67000 Kw a T_2 e 1110000 Kw a T_3.
C_2 invia 99000 Kw T_1, 1000 Kw a T_2 e 100000 Kw a T_3.
C_3 invia 499000 Kw T_1, 1000 Kw a T_2 e 100000 Kw a T_3.
C_4 invia 299000 Kw T_1, 1000 Kw a T_2 e 100000 Kw a T_3.