Introduction

Insulating cavity in a conductor.

Strategy for uniqueness.

Tools for stability.

Cavity with boundary impedance.

Rigid inclusion in an elastic body.

End.

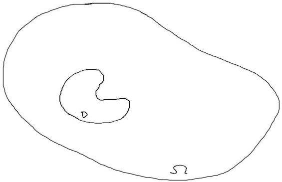
Inverse problems with unknown boundaries: uniqueness and stability

Giovanni Alessandrini[1]

[1] DMI Università di Trieste

Cartagena, PICOF 2010

A family of problems



Consider a body $\Omega \subset \mathbb{R}^n$ which might contain an unknown, inaccessible, cavity D (or an inclusion). To detect the presence and the shape of D from measurements taken from the exterior, accessible, boundary of Ω, when some field (electric, electromagnetic, thermal, elastic, . . .) is applied to it.

A family of problems

- Ω electrical conductor, D cavity with insulating boundary,
- Ω electrical conductor, D perfectly conducting inclusion,

Andrieux, Ben Abda, Jaoua (1993), Beretta, Vessella (1996), Bukhgeim, Cheng, Yamamoto (1998, 1999, 2000) Cheng, Hon and Yamamoto (2001), A., Beretta, Rosset, Vessella (2000), A., Rondi (2001).

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Higashimori (2002), Morassi, Rosset (2004).

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Alvarez, Conca, Friz, Kavian, Ortega (2005), Doubova, Fernández-Cara, González-Burgos, Ortega (2006), Doubova, Fernández-Cara, Ortega (2007), Ballerini (2010).

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The prototype.

insulating cavity in a conductor

Assume $\Omega \setminus \overline{D}$ connected.

$$\begin{cases} \Delta u = 0, & \text{in} \quad \Omega \setminus \overline{D}, \\ \nabla u \cdot \nu = 0, & \text{on} \quad \partial D, \\ \nabla u \cdot \nu = \psi, & \text{on} \quad \partial \Omega. \end{cases}$$

ν exterior unit normal to $\partial(\Omega \setminus \overline{D})$. $\int_{\partial\Omega} \psi = 0$.

Find D given $u|_{\partial\Omega}$.

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instability

A.–Rondi (2001). Let $n = 2$, $\Omega = B_1(0)$, $D_0 = B_{1/2}(0)$, denote $z = x + iy$ and

$$f_k(z) = z \exp[\epsilon_k(z^k - z^{-k})], z \neq 0,$$

with

$$\epsilon_k = O(k^{-M}2^{-k}) \in \mathbb{R}, \ k = 1, 2, \dots$$

denote $D_k = f(D_0)$. Then D_k are uniformly C^M-smooth and

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$$d_{\mathcal{H}}(\partial D_0, \partial D_k) \sim k^{-M} \to 0 \text{ polynomially },$$

whereas,

letting u_k be the potential corresponding to D_k, $k = 0, 1, \ldots$

$$\|u_k - u_0\|_{L^2(\partial\Omega)} \sim \epsilon_k^{1/2} \to 0 \text{ exponentially }.$$

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Strategy for uniqueness.

Given two cavities D_1, D_2, and given a nontrivial boundary current density ψ, let u_1, u_2 solve for $i = 1, 2$

$$\begin{cases} \Delta u_i = 0, & \text{in} \quad \Omega \setminus \overline{D_i}, \\ \nabla u_i \cdot \nu = 0, & \text{on} \quad \partial D_i, \\ \nabla u_i \cdot \nu = \psi, & \text{on} \quad \partial \Omega, \end{cases}$$

and suppose D_1, D_2 give rise to the same potential on $\partial\Omega$: $u_1|_{\partial\Omega} = u_2|_{\partial\Omega}$.
If we had $D_1 \neq D_2$,we might assume w.l.o.g. $D_2 \setminus \overline{D_1} \neq \emptyset$.

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Strategy for uniqueness.

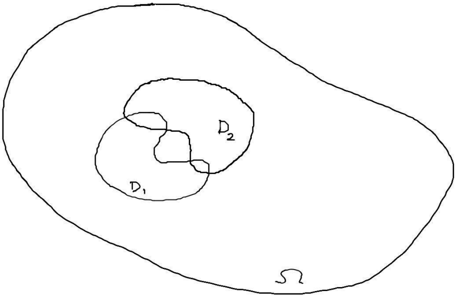

Figure: two cavities.

Strategy for uniqueness.

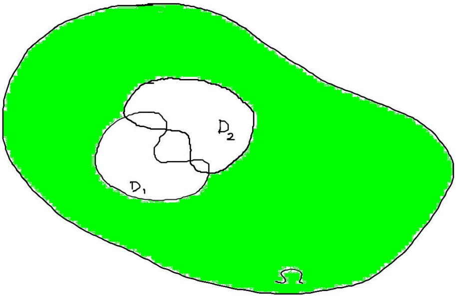

Figure: the set G.

G: connected component of $\Omega \setminus (\overline{D_1 \cup D_2})$ such that $\partial\Omega \subset \partial G$.

Strategy for uniqueness.

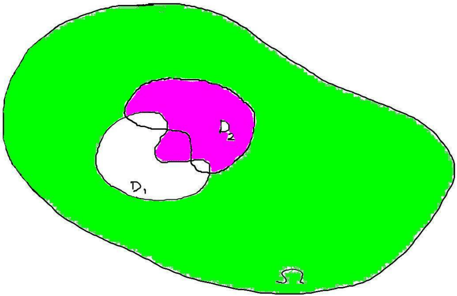

Figure: the set $E_2 \supset D_2 \setminus \overline{D_1}$.

$$E_2 = \Omega \setminus \overline{D_1 \cup G}$$
$$\partial E_2 = \Gamma_1 \cup \Gamma_2, \Gamma_1 \subset (\partial D_1 \setminus G), \Gamma_2 \subset (\partial D_2 \cap \partial G).$$

Strategy for uniqueness.

u_1, u_2 have the same Cauchy data on $\partial\Omega$:

$$\nabla u_1 \cdot \nu = \nabla u_2 \cdot \nu = \psi \text{ and } u_1 = u_2 \text{ on } \partial\Omega.$$

Hence, by unique continuation,

$$u_1 \equiv u_2 \text{ in } G,$$

$$\Downarrow$$

$$\nabla u_1 \cdot \nu = \nabla u_2 \cdot \nu = 0 \text{ on } \Gamma_2 \subset (\partial D_2 \cap \partial G)$$

Therefore

$$\int_{D_2 \setminus \overline{D_1}} |\nabla u_1|^2 \le \int_{E_2} |\nabla u_1|^2 \le$$

$$\le \int_{\Gamma_1} |u_1 \nabla u_1 \cdot \nu| + \int_{\Gamma_2} |u_1 \nabla u_2 \cdot \nu| = 0.$$

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Strategy for uniqueness.

Either:

$u_1 \equiv$ constant on an open set

unique continuation

$\Downarrow$

$$\psi \equiv 0,$$

or

$$D_2 \setminus \overline{D_1} = \emptyset.$$

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Tools for stability.

- Assume a-priori $C^{1,\alpha}$ bounds on $\partial D_1, \partial D_2$ and on $\partial\Omega$.
- Assume ψ nontrivial:

$$\frac{\|\psi\|_{L^2(\partial\Omega)}}{\|\psi\|_{H^{-1/2}(\partial\Omega)}} \leq F.$$

- Assume

$$\|u_1 - u_2\|_{L^2(\partial\Omega)} \leq \epsilon.$$

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Tools for stability.

step 1

Stability for a Cauchy problem in G.

$$\int_{D_2\setminus\overline{D_1}} |\nabla u_1|^2 \leq \omega^{(2)}(\epsilon)$$

where $\omega^{(2)}(\epsilon) = \omega \circ \omega(\epsilon)$ and

$$\omega(\epsilon) \sim |\log \epsilon|^{-\gamma}, \text{ as } \epsilon \to 0.$$

Improved stability for a Cauchy problem in G.
If in addition, G is known to be Lipschitz, then

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Tools for stability.

step 2

Propagation of smallness.

If, for a suitable $s > 1$, $B_{s\rho}(x) \subset \Omega \setminus \overline{D_1}$ then

$$\int_{B_\rho(x)} |\nabla u_1|^2 \geq \frac{C(F)}{\exp[A\rho^{-B}]} \int_{\Omega\setminus\overline{D_1}} |\nabla u_1|^2 .$$

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step 3

Geometric argument.

$$inradius(D_2 \setminus \overline{D_1}) + inradius(D_1 \setminus \overline{D_2}) \leq \omega^{(3)}(\epsilon)$$

using the $C^{1,\alpha}$ a-priori bound

$$d_{\mathcal{H}}(\partial D_1, \partial D_2) \leq \omega^{(3)}(\epsilon).$$

When ϵ is small enough, then the above rough bound implies that G is Lipschitz, we can use the improved estimate for the Cauchy problem and arrive at

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Tools for stability.

step 4

How to improve the propagation of smallness?

Doubling at the boundary, with Neumann condition.

Adolfsson and Escauriaza (1997).

If $\partial D_1 \in C^{1,1}$ then $\forall x \in \partial D_1$

$$\int_{B_{2\rho}\setminus\overline{D_1}} |\nabla u_1|^2 \leq C(F) \int_{B_\rho\setminus\overline{D_1}} |\nabla u_1|^2$$

$$\Downarrow$$

$$\int_{B_\rho(x)\setminus\overline{D_1}} |\nabla u_1|^2 \geq C\rho^K \int_{\Omega\setminus\overline{D_1}} |\nabla u_1|^2.$$

with $C, K > 0$ depending on F.

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$$\int_{B_{2\rho}\setminus\overline{D_1}} |\nabla u_1|^2 \leq C(F) \int_{B_\rho\setminus\overline{D_1}} |\nabla u_1|^2$$

$$\Downarrow$$

$$\int_{B_\rho(x)\setminus\overline{D_1}} |\nabla u_1|^2 \geq C\rho^K \int_{\Omega\setminus\overline{D_1}} |\nabla u_1|^2.$$

with $C, K > 0$ depending on F.

Tools for stability.

step 4

How to improve the propagation of smallness?
Doubling at the boundary, with Neumann condition.
Adolfsson and Escauriaza (1997).
If $\partial D_1 \in C^{1,1}$ then $\forall x \in \partial D_1$

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conclusion

In summary:
we obtain

$$d_{\mathcal{H}}(\partial D_1, \partial D_2) \le \omega^{(2)}(\epsilon).$$

using

stability for the Cauchy pb. and propagation of smallness

$\Uparrow$

three spheres inequality

If we also have the

doubling inequality at the boundary

then we arrive at

$$d_{\mathcal{H}}(\partial D_1, \partial D_2) \le \omega(\epsilon).$$

A., Beretta, Rosset, Vessella (2000).

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the three spheres inequality

For every $0 < r_1 < r_2 < r_3$

$$\int_{B_{r_2}} |u|^2 \leq C \left(\int_{B_{r_1}} |u|^2 \right)^{\alpha} \left(\int_{B_{r_3}} |u|^2 \right)^{1-\alpha}$$

with $C > 0, 0 < \alpha < 1$ only depending on $\frac{r_2}{r_1}, \frac{r_3}{r_2}$.

Cavity with impedance.

$$\begin{cases} \Delta u = 0, & \text{in} \quad \Omega \setminus \overline{D}, \\ \nabla u \cdot \nu + \gamma u = 0, & \text{on} \quad \partial D, \qquad \gamma \geq 0 \\ \nabla u \cdot \nu = \psi, & \text{on} \quad \partial \Omega. \end{cases}$$

ν exterior unit normal to $\partial(\Omega \setminus \overline{D})$.

- Non-uniqueness: one pair of Cauchy data $(\psi, u|_{\partial\Omega})$ does not suffice to uniquely determine D (and γ). Cakoni, Kress (2007), Rundell (2008).
- Uniqueness: two pairs of Cauchy data $(\psi, u|_{\partial\Omega})$ and $(\widetilde{\psi}, \widetilde{u}|_{\partial\Omega})$, with linearly independent $\psi, \widetilde{\psi}$ and $\psi \geq 0$ uniquely determine D and γ. Bacchelli (2009), Pagani, Pierotti (2009).
- Stability: with two such pairs there is *log*-stability. Sincich (2010).

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Cavity with impedance.

what goes wrong?

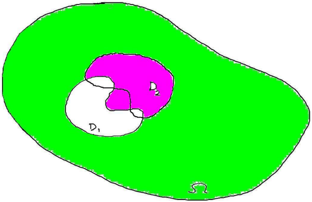

Figure: the set $E_2 \supset D_2 \setminus \overline{D_1}$.

$$
\begin{cases}
\Delta u_1 = 0, & \text{in} \quad E_2, \\
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-\nabla u_1 \cdot \nu + \gamma_2 u_1 = 0, & \text{on} \quad \partial E_2 \cap \partial D_2,
\end{cases}
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ν exterior unit normal to E_2.

Cavity with impedance.

the approach by Sincich

Let u_i be the potential corresponding to D_i, $i = 1, 2$.
If $\psi \gneq 0$ then (strong maximum principle) $u_i > 0$.
Set

$$v_i = \frac{\widetilde{u}_i}{u_i},$$

then

$$\begin{cases} \operatorname{div}(u_i^2 \nabla v_i) = 0, & \text{in} \quad \Omega \setminus \overline{D_i}, \\ u_i^2 \nabla v_i \cdot \nu = 0, & \text{on} \quad \partial D_i, \\ u_i^2 \nabla v_i \cdot \nu = u_i \widetilde{\psi} - \widetilde{u}_i \psi, & \text{on} \quad \partial \Omega. \end{cases}$$

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Cavity with impedance.

open problem

What if both $\psi, \widetilde{\psi}$ change sign?

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Rigid inclusion in elastic body.

In $\mathbb{R}^3$ (or $\mathbb{R}^2$).

$$\begin{cases} \mathrm{div}(\mu(\nabla u + \nabla u^T)) + \nabla(\lambda \mathrm{div} u) = 0, & \text{in} \quad \Omega \setminus \overline{D}, \\ u \in \mathcal{R}, & \text{on} \quad \partial D, \\ (\mu(\nabla u + \nabla u^T) + (\lambda \mathrm{div} u) I)\nu = \psi, & \text{on} \quad \partial\Omega. \end{cases}$$

Lamè parameters $\mu, \lambda \in C^{1,1}$ satisfying strong convexity $\mu \geq \alpha > 0$, $2\mu + 3\lambda \geq \beta > 0$.

$$\mathcal{R} = \text{ space of infinitesimal rigid displacements } = \{r(x) | r(x) = c + Wx, c \in \mathbb{R}^3, W + W^T = 0\}$$

+ equilibrium condition

$$\int_{\partial D} (\mu(\nabla u + \nabla u^T) + (\lambda \mathrm{div} u) I)\nu \cdot r = 0 \; \forall r \in \mathcal{R}$$

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Inverse problem: given $u|_{\partial\Omega}$ find D.

Morassi and Rosset (2009): uniqueness and $log - log$ stability.

Let u_i be the displacement field corresponding to $D_i, i = 1, 2$, we have $u_i = r_i \in \mathcal{R}$, with r_i unknown possibly different.

Rigid inclusion in elastic body.

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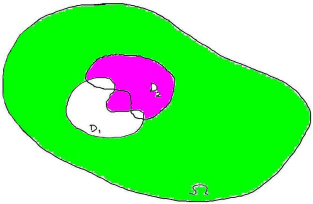

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Two cases

1. $\partial E_2 \cap \partial D_1 \cap \partial D_2$ contains at least three points not aligned.
2. $\partial E_2 \cap \partial D_1 \cap \partial D_2 \subset$ segment.

1. $r_1 = r_2 \Rightarrow u_1 \equiv r_2$ in E_2.
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Unknown Boundaries

Giovanni Alessandrini

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Doubling at the boundary?

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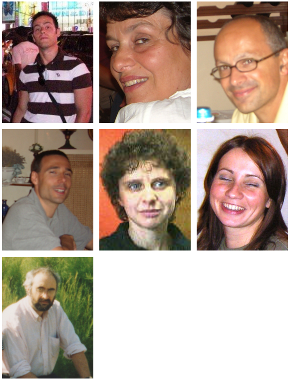
My collaborators.



Andrea Ballerini, Elena Beretta, Antonino Morassi,
Luca Rondi, Edi Rosset, Eva Sincich,
Sergio Vessella.

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The end.

THANKS!