INdAM Workshop

Inverse Problems and Applications

June 3–9, 2002

Cortona, Italy

Organizers

Giovanni Alessandrini (Università degli Studi di Trieste, Italy)

Gunther Uhlmann (University of Washington, USA)

Program

Monday June 3, 2002

10:00 – 10:45
F. Alberto Grünbaum (University of California, Berkeley, USA)
Diffuse or optical tomography as a general nonlinear inverse problem for a Markov chain

11:15 – 12:00
Erkki Somersalo (Helsinki University of Technology, Finland)
Inverse boundary value problem for Maxwell equations with scalar impedance

12:15 – 13:00
Sergio Vessella (Università degli Studi di Firenze, Italy)
Optimal three cylinder inequality for solutions to parabolic equations and stability estimates in inverse problems

Tuesday June 4, 2002

9:00 – 9:45
ALFREDO LORENZI (Università degli Studi di Milano, Italy)
Identification problems for linear integro-differential equations

10:00 – 10:45
DAVID ISAACSON (Rensselaer Polytechnic Institute, New York, USA)
Electrical Impedance Imaging for Medical Applications

11:15 – 12:00
MAARTEN V. DE HOOP (Colorado School of Mines, Golden CO, USA)
Seismic inverse scattering in the 'wave-equation' approach

12:15 – 13:00
MATTI LASSAS (University of Helsinki, Finland)
Inverse boundary value problems on non-compact manifolds

Short Communications

15:35 – 15:55
ALEXANDRU TAMASAN (University of Washington, USA)
2D Optical Tomography in Critical Regime

16:30 – 16:50
GABRIELE INGLESE (Istituto di Analisi Globale ed Applicazioni, CNR, Florence, Italy)
An algorithm for corrosion detection via simultaneous reconstruction of material loss and energy dispersion coefficients

16:55 – 17:15
BRUNO CANUTO (Université Blaise Pascal, France)
Determining Two Coefficients in Elliptic Operators via Boundary Spectral Data: a Uniqueness Result

Wednesday June 5, 2002

9:00 – 9:45
Michael Vogelius (Rutgers, The State University of New Jersey, USA)
Blow-up of solutions to non-linear elliptic Neumann problems

10:00 – 10:45
Plamen Stefanov (Purdue University, USA)
Inverse Problem for the 2D transport equation

11:15 – 12:00
Mauro Giudici (Università degli Studi di Milano, Italy)
Some problems for the application of inverse techniques to environmental modelling

12:15 – 13:00
Liliana Borcea (Rice University, USA)
Optimal finite difference grids for direct and inverse Sturm Liouville problems

Thursday June 6, 2002

9:00 – 9:45
Victor Isakov (Wichita State University, USA)
On uniqueness in the lateral Cauchy problem and on inverse problems for the dynamical Lame system

10:00 – 10:45
Fadil Santosa (University of Minnesota, USA)
Inverse Problems Seeking Solutions

11:15 – 12:00
Allan Greenleaf (University of Rochester, USA)
Global uniqueness for conormal conductivities and potentials

12:15 – 13:00
Elena Beretta (Università degli Studi di Roma "La Sapienza", Italy)
Asymptotic expansions of the steady-state voltage potential in the presence of thin inhomogeneities

Short Communications

15:35 – 15:55
Peter Mathé (Weierstrass Institute Berlin, Germany)
Optimal Discretization of Inverse Problems in Hilbert Scales

16:30 – 16:50
Alberto Favaron (Università degli Studi di Milano, Italy)
Parabolic integrodifferential identification problems related to memory kernels with special symmetries

16:55 – 17:15
Elisa Francini (Istituto di Analisi Globale ed Applicazioni, CNR, Florence, Italy)
Asymptotic formulas for perturbations in the electromagnetic fields due to the presence of thin inhomogeneities

Friday June 7, 2002

9:00 – 9:45
Lassi Päivärinta (University of Oulu, Finland)
Inverse scattering from a random potential in 2D

10:00 – 10:45
Yaroslav V. Kurylev (Loughborough University, UK)
Stability in Inverse Boundary Spectral Problems

11:15 – 12:00
Clifford Nolan (Rensselaer Polytechnic Institute, New York, USA)
Microlocal Methods for Anisotropic Materials

12:15 – 13:00
Antonino Morassi (Università degli Studi di Udine, Italy)
Detecting inclusions with extreme conductivities

Saturday June 8, 2002

9:00 – 9:45
Mario Bertero (Università di Genova, Italy)
Restoration of astronomical images at thermal infrared wavelenghts

10:00 – 10:45
William Rundell (Texas A&M University, USA)
Some new inverse eigenvalue problems

11:15 – 12:00
Gunther Uhlmann (University of Washington, USA)
Travel time tomography

Abstracts

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ELENA BERETTA

Università degli Studi di Roma "La Sapienza", Italy

Asymptotic expansions of the steady-state voltage potential in the presence of thin inhomogeneities

Joint work with E. Francini and M. Vogelius

We derive rigorously asymptotic expansions for the steady-state voltage potential in the presence of a finite number of thin inhomogeneities. We use these formulae to establish uniqueness results in the identification of the inhomogeneities from boundary measurements.

Mario Bertero

Università di Genova, Italy

Restoration of astronomical images at thermal infrared wavelenghts

Joint work with P. Boccacci, Università di Genova, Italy

Ground-based astronomical observations at thermal infrared wavelengths (around 10-20 microns) face the problem of extracting the weak astronomical signal from the large background due to atmosphere and telescope emission. To this purpose the acquisition technique known as "chopping and nodding" is used. The resulting image can be modeled as the second difference of the image of the target and is affected by large negative counterparts of the sources. In this paper we propose a constrained iterative method for the restoration of the original image and we test its performance by means of numerical simulations.

Liliana Borcea

Rice University, USA

Optimal finite difference grids for direct and inverse Sturm Liouville problems

Work in collaboration with Vladimir Druskin, Schlumberger Doll

We study finite difference approximations of solutions of direct and inverse Sturm Liouville problems, in a finite or infinite interval on the real line. The discretization is done on optimal grids, with a three-point finite difference stencil. The optimal location of the grid points is calculated via a rational approximation of the Neumann to Dirichlet map and the latter converges exponentially fast. We prove that optimal grids obtained for constant coefficients are asymptotically optimal for variable coefficient direct problems. We also show that optimal grids, together with methods of inverse spectral problems for Jacobi matrices, can be used for the solution of continuous inverse Sturm Liouville problems. In particular, we formulate and analyze a new inversion algorithm, where the unknown coefficients that we image are optimally discretized. We prove that optimal grids provide necessary conditions for convergence of the discrete inverse problem and we demonstrate the effectiveness of our imaging approach through numerical simulations.

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Bruno Canuto
Université Blaise Pascal, France

Determining Two Coefficients in Elliptic Operators via Boundary Spectral Data: a Uniqueness Result

Joint work with Otared Kavian, Université de Versailles, France

For a bounded and sufficiently smooth domain Ω in $\mathbb{R}^N$, $N \geq 2$, let $(\lambda_k)_{k=1}^{\infty}$ and $(\varphi_k)_{k=1}^{\infty}$ be respectively the eigenvalues and the corresponding eigenfunctions of the problem (with Neumann boundary conditions)

$$-\mathrm{div}(a(x)\nabla\varphi_k) + q(x)\varphi_k = \lambda_k \rho(x)\varphi_k \quad \text{in } \Omega, \quad a\frac{\partial}{\partial \mathbf{n}}\varphi_k = 0 \quad \text{on } \partial\Omega.$$

We prove that knowledge of the Dirichlet boundary spectral data $(\lambda_k)_{k=1}^{\infty}$, $(\varphi_{k|\partial\Omega})_{k=1}^{\infty}$ determines uniquely the Neumann-to-Dirichlet (or the Steklov-Poincaré) map γ for a related elliptic problem. Under suitable hypothesis on the coefficients a, q, ρ their identifiability is then proved. We prove also analogous results for Dirichlet boundary conditions.

Maarten V. de Hoop
Colorado School of Mines, Golden CO, USA

Seismic inverse scattering in the 'wave-equation' approach

This research was carried out in collaboration with C.C. Stolk and J.L. Le Rousseau. Part of this research was carried out at the Mathematical Sciences Research Institute, Berkeley CA, USA

In reflection seismology one places point sources and point receivers on the Earth's surface. The source generates acoustic waves in the subsurface, that are reflected where the medium properties vary discontinuously. The recorded reflections that can be observed in the data are used to reconstruct these discontinuities.

Seismic data are commonly modeled by a high-frequency single scattering approximation. This amounts to a linearization in the medium coefficient about a smooth background. The discontinuities are contained in the medium perturbation. Both the smooth background and the perturbation are in general unknown and have to be reconstructed jointly. The wave solutions in the background medium admit a geometrical optics representation. Here we describe the wave propagation in the background medium by a one-way wave equation. Based on this we derive the double-square-root equation, which is a first-order pseudodifferential equation, that describes the continuation of seismic data in depth. We consider the modeling operator, its adjoint and reconstruction based on this equation in the framework of microlocal analysis. If the rays in the background that are associated with the reflections due to the perturbation are nowhere horizontal, the singular part of the data is described by the solution to an inhomogeneous double-square-root equation. We derive a microlocal reconstruction equation for the medium perturbation. Finally, pseudodifferential annihilators based on the double-square-root equation are constructed. These annihilators form the criterion to determine the smooth background medium. We illustrate the mentioned inverse scattering procedure in the so-called generalized screen approximation which follows an expansion of the symbol of the one-way wave operator into elementary symbols.

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ALBERTO FAVARON
Università degli Studi di Milano, Italy

Parabolic integrodifferential identification problems related to memory kernels with special symmetries

Joint work with Alfredo Lorenzi (Università degli Studi di Milano, Italy)

The aim of this paper is to generalize our previous results obtained in [1] to the case of the generalized corona $\Omega_\mu = \{x{=}(x_1, x_2, x_3) \in \mathbb{R}^3 : c\mu(|x|^{-1}x) < |x| < \mu(|x|^{-1}x)\}$, where $0 < c < 1$, $|x| = (x_1^2 + x_2^2 + x_3^2)^{1/2}$ and $\mu \in C^2(\mathcal{O})$, $\mathcal{O}$ being an open set containing $\partial B(0,1)$. We note that the two disjoint surfaces $|x| = c\mu(|x|^{-1}x)$ and $|x| = \mu(|x|^{-1}x)$, which the boundary $\partial\Omega_\mu$ consists of, are related by a dilatation.
We are interesting in identifying the unknown memory kernel $k(t, \rho)$, appearing in the following integro-differential equation of parabolic type

$$D_t u(t,x) = \mathcal{A}u(t,x) + \int_0^t k\Big(t-s, \frac{|x|}{\mu(|x|^{-1}x)}\Big)\mathcal{B}u(s,x)\,\mathrm{d}s + \int_0^t D_\rho k\Big(t-s, \frac{|x|}{\mu(|x|^{-1}x)}\Big)\mathcal{C}u(s,x)\,\mathrm{d}s \,+\, f(t,x), \quad \forall\,(t,x) \in [0,T]\times\Omega_\mu. \qquad (1)$$

In equation (1) $\mathcal{A}$ and $\mathcal{B}$ are two second-order linear differential operators, $\mathcal{A}$ being uniformly elliptic and in divergence form, while $\mathcal{C}$ is a first-order differential operator.
Though our identification problem seems to be a simple generalization to the case of a generalized corona Ω_μ of the one dealt with in [1] related to a *spherical* corona Ω_1 the situation is much more complex.
We will single out a special class of *admissible* operators $\mathcal{A}$ and two pieces of suitable additional information for which the problem of identifying k can be uniquely solved locally in time at least when the domain Ω_μ is a *small* deformation of the spherical corona Ω_1.

[1] Favaron A., Lorenzi A.: *Parabolic integrodifferential equations related to radial memory kernels I,* J. Inverse Ill Posed Problems, 9 (2001), 489–529.

Elisa Francini

Istituto di Analisi Globale ed Applicazioni, CNR, Florence, Italy

Asymptotic formulas for perturbations in the electromagnetic fields due to the presence of thin inhomogeneities

Joint work with E. Beretta and M. Vogelius

We consider solutions to the time-harmonic Maxwell's Equations of a TE nature. For such solutions we provide a rigorous derivation of the first order boundary perturbation caused by the presence of thin inhomogeneities.

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Mauro Giudici

Università degli Studi di Milano, Italy

Some problems for the application of inverse techniques to environmental modelling

One of the most important steps in the development and application of an environmental model is the model calibration, i.e. the determination of the values of the model parameters that permit the best fitting between model results and measured data. This is a typical example of inverse problem. At every phase of the model development and application, we must consider the goals of the model and the (space and time) scales at which the phenomena are described by the model. These aspects have strong impact on the solution to the inverse problem.

In particular the choice of the model scales is of paramount importance, because it controls the ability of the model to reproduce real features and the discretization of the domain for the numerical model. Moreover, a necessary condition to be confident on model forecast is that the model scales are consistent with the space and time distribution of the field measurements. Decreasing the grid spacing can improve the ability of the model to reproduce fine-scale features. However, this requires a detailed knowledge of the physical system, which could be a difficult task if the natural system is heterogeneous or anisotropic, as is often the case for geological formations or turbulent flows. Therefore the modeler of the physical system must work with numerical techniques that provide "good solutions" even for grids with "large spacing". "Good solutions" are solutions which satisfy the physical principles that are at the base of the model, i.e. the conservation principles. "Large spacing" means that (1) the grid elements or cells could be larger than the optimal value coming from the purely mathematical theory, but consistent with the distribution of the available measurements and (2) the grid elements could be larger than the volume over which phenomenological laws are validated with laboratory or field experiments.

These remarks are important for model calibration and therefore for the solution to the inverse problem, above all in the discrete case, which is the most important for applications. In particular, stability is usually an overwhelming problem for continuous domains, but numerical instability is often linked to ill conditioning. Moreover, ill conditioning depends upon the model scale, which is strictly linked to the grid spacing and the spatial data distribution.

Furthermore, in the last decades, several works, dealing with both statistical and deterministic inverse techniques, have shown the importance of using several independent data sets to reduce uncertainty in the model. This is very important because, in principle, the parameters of the discrete models are non local quantities, in the sense that they depend upon the space distribution in a region wider than the area to which the parameters refers to and on the flow direction. Therefore, parameters obtained from data sets corresponding to different flow situations are more confident and, as a consequence, this improves the reliability of model forecasting.

Allan Greenleaf

University of Rochester, USA

Global uniqueness for conormal conductivities and potentials

Joint work with Matti Lassas and Gunther Uhlmann

We consider global uniqueness and reconstruction in the Calderon problem for some classes of conductivities which do not lie in the currently best known spaces for this problem. The conductivities (and potentials) that we consider are conormal with respect to (possiby disconnected) submanifolds compactly contained in the domain, in three or more dimensions. Examples in this category include conductivities which are only $C^{1+\epsilon}$.

F. Alberto Grünbaum
University of California, Berkeley, USA

Diffuse or optical tomography as a general nonlinear inverse problem for a Markov chain

I pose the problem of recovering the one step transition probability matrix from "boundary measurements" for a discrete state space Markov chain with three kinds of states" incoming, outgoing, and (most of them) hidden states. One allows for absorption too and in the case of continuous time the (Poisson) rates for the holding time at each hidden state can also be considered as unknown. This is a very unrealistic model of optical tomography but may have applications in other "network tomography" situations.

Gabriele Inglese

Istituto di Analisi Globale ed Applicazioni, CNR, Florence, Italy

An algorithm for corrosion detection via simultaneous reconstruction of material loss and energy dispersion coefficients

Joint work with Dario Fasino (Università degli Studi di Udine, Italy)

We derive an algorithm for the reconstruction of a pair of coefficients that appear in the description of corrosion damages over the top surface (supposed to be inaccessible) of a thin homogeneous metallic plate. Moreover, we suppose that the plate is coated with an insulating film. Laplace's equation holds inside the object and Cauchy data sets can be collected on the bottom side of the plate. We propose the simple physical model in which the first coefficient corresponds to the material loss (boundary identification) while the second one is the transfer coefficient in a Robin boundary condition modeling the energy dispersion due to the local degrade of the insulating film. We expand all the functions in the problem in powers of the width of the plate and produce a Thin Plate Approximation of the two corrosion coefficients. We determine a range of values in which the method is succesful.

DAVID ISAACSON

Rensselaer Polytechnic Institute, New York, USA

Electrical Impedance Imaging for Medical Applications

Electrical Impedance Imaging systems apply currents to a bodys surface and measure the voltages that result. From this boundary data they reconstruct and display images of the electrical conductivity and permittivity inside the body.

It will first be explained how the design of these systems requires an analysis of inverse problems for Maxwell's equations.

It will next be explained how these images are made and how they are used for monitoring and diagnosing heart disease , lung function , and breast cancer.

Images and movies made by the Adaptive Current Tomography systems at RPI will be shown.

Victor Isakov

Wichita State University, USA

On uniqueness in the lateral Cauchy problem and on inverse problems for the dynamical Lame system

A part of these results is obtained jointly with Oleg Imanuvilov and Masahiro Yamamoto

We give Carleman type estimates for the dynamical Lame system and derive from them uniqueness of the continuation results in the lateral Cauchy problem and in the problem of identification of all three elastic parameters from few and many boundary observations. An important tool is Carleman type estimates in negative Sobolev spaces.

Yaroslav V. Kurylev

Loughborough University, UK

Stability in Inverse Boundary Spectral Problems

Joint work with A. Katsuda and M. Lassas

We consider conditions upon the geometry of manifolds which guarantee their precompactness in the Gromov-Hausorff topology. In particular, we search for those which would guarantee stability in the Gel'fand inverse boundary spectral problem.

Matti Lassas

University of Helsinki, Finland

Inverse boundary value problems on non-compact manifolds

The results concerning elliptic equation are done in collaboration with M. Taylor and G. Uhlmann and the results for hyperbolic equations are obtained with A. Katchalov and Y. Kurylev

In applications many inverse boundary value problems are formulated in unbounded domains, for instance in half-space. Because inverse problem for anisotropic equations are often formulated using Riemannian manifolds we discuss different inverse boundary value problems on non-compact, complete manifolds. On these manifolds we consider inverse problems for elliptic and hyperbolic equations.

For the elliptic equations we formulate result for real-analytic manifolds and give in 2-dimensional case examples of non-homeomorphic manifolds that Cauchy data of harmonic functions coincide.

For hyperbolic inverse problems we consider smooth (non-real analytic) manifolds and consider different types of boundary data. For instance we consider the following measurements:

1. Assume that we know the Cauchy data of all solutions.

2. Assume that we know how much energy one has to use to force the solution to have given the boundary values.

3. Assume that we know boundary values of Schwartz kernel of spectral projections.

We show that all these data are equivalent and show that any of these data is enough to solve the inverse problem.

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Alfredo Lorenzi
Università degli Studi di Milano, Italy

Identification problems for linear integro-differential equations

The talk will be concerned with some of the following topics:

1. recovering some *scalar* differentiable relaxation kernels in the integro-differential Maxwell equations for dispersive media with cylindrical symmetries;

2. recovering the *scalar* differentiable relaxation kernel h in second-order linear integro-differential operator equations of the form

$$u''(t) - \int_0^t h(t-s)Au(s)\,\mathrm{d}s = f(t), \qquad t \in [0, +\infty),$$

   in the so-called "singular case", corresponding to $h(0) = 0$;

3. recovering the conductivity coefficient dependent only on time and a memory kernel in parabolic integro-differential equations.

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Peter Mathé

Weierstrass Institute Berlin, Germany

Optimal Discretization of Inverse Problems in Hilbert Scales

We study the efficiency of the approximate solution of ill-posed problems, based on discretized noisy observations, which we assume to be given beforehand. We restrict ourselves to problems which can be formulated in Hilbert scales. Within this framework we shall quantify the degree of ill-posedness, provide general conditions on projection schemes to achieve the best possible order of accuracy. Coditions are given in terms of approximation properties (Bernstein and Jackson type inequalities) of the involved projections. We pay particular attention on the problem of self-regularization vs. Tikhonov regularization.

Antonino Morassi

Università degli Studi di Udine, Italy

Detecting inclusions with extreme conductivities

We prove upper and lower bounds on the size of an unknown cavity in an electrical conductor in terms of boundary measurements of voltage and current.

Clifford Nolan

Rensselaer Polytechnic Institute, New York, USA

Microlocal Methods for Anisotropic Materials

Microlocal analysis has proven itself a valuable tool in the modeling of wave propagation in homogeneous and isotropic elastic and electromagnetic materials. This approach to modeling of such waves has also yielded successful methods in solving the associated inverse problems of estimating material parameters, such as elastic moduli, electrical permittivity, etc. In this talk, we will examine how microlocal analysis may be used to model wave propagation in anisotropic materials and the implications for the inverse problems.

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Lassi Päivärinta
University of Oulu, Finland

Inverse scattering from a random potential in 2D

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William Rundell

Texas A&M University, USA

Some new inverse eigenvalue problems

We will look at two quite different problems to recover the potential in a second order differential operator from information on its spectrum. The first concerns the classical equation $-u'' + qu = \lambda u$ with (say) the boundary condition $u(0) = 0$. At the other end point, $x = 1$ $u'(1) = \sqrt{\lambda}u(1)$. This problem has complex eigenvalues and we will show that a single such spectrum suffices to determine q. In the second problem we have the equation $-u'' + \ell(\ell+1)/x^2 + qu = \lambda u$ with fixed conditions at $x = 1$ and boundedness at $x = 0$. We examine the conjecture that two complete spectra $\{\lambda_{n,\ell}\}$ for $n = 1, 2, \ldots$ and for ℓ equals two distinct values ℓ_1, ℓ_2 is sufficient to determine q.

Fadil Santosa
University of Minnesota, USA

Inverse Problems Seeking Solutions

Inverse problems arise in many industrial applications. I would like to take this opportunity to present three problems whose solution will be of great interest to industry. All the problems are to a great extent unsolved.

1. Design of shapes with prescribed pressure profiles (from the auto industry)
2. Characterization of fiber optical waveguides (from an optical communication company)
3. Nondestructive evaluation of spot welds (also from the auto industry)

In each case, I will start with a description of the model, what is measured or given, and what is to be determined. Specific mathematical questions will be posed. The objective of this presentation is to start new mathematical research in these important applications.

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Erkki Somersalo

Helsinki University of Technology, Finland

Inverse boundary value problem for Maxwell equations with scalar impedance

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Plamen Stefanov

Purdue University, USA

Inverse Problem for the 2D transport equation

We prove uniqueness result and a stability estimate for the 2D transport equation about the recovery of the collision kernel $k(x, v'v)$ and the absorption coefficient $\sigma(x)$ from the albedo operator on the boundary. This is a formally determined problem for the recovery of k and we assume that k is small enough.

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Alexandru Tamasan
University of Washington, USA

2D Optical Tomography in Critical Regime

Optical tomography is a potential imaging method in medical diagnosis, which uses near-infrared radiation. The mathematical model is based on the transport equation

$$\theta \cdot \nabla u(x,\theta) + a(x)u(x,\theta) = \int_{\mathbf{S}^1} k(x,\theta\cdot\theta')u(x,\theta')d\theta', \; x\in\Omega, \; \theta\in\mathbf{S}^1.$$

When no collided particles get absorbed, the balance between the total cross section and scatter is given by $\int_{\mathbf{S}^1} k(x,\theta\cdot\theta')d\theta' = a(x)$. When scattering is large, the flux u is near linear in θ. A linearity assumption reduces the transport equation to a diffusion equation. Moreover, knowledge of the albedo operator (boundary data operator) translates into knowledge of the Dirichlet-to-Neumann map. The $\bar{\partial}$-method applied to the diffusion equation recovers $k_1(x) = \int_{\mathbf{S}^1} \theta\cdot\theta' k(x,\theta\cdot\theta')d\theta$.

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Gunther Uhlmann
University of Washington, USA

Travel time tomography

We survey recent results on the inverse kinematic problem The question is whether one can determine the sound speed (index of refraction) of a medium by measuring the travel times of the corresponding ray paths.

This inverse problem arose in geophysics in an attempt to determine the substructure of the Earth by measuring at the surface of the Earth the travel times of seismic waves. An early success of this inverse method was the estimate by Herglotz and Wiechert and Zoeppritz of the structure of the Earth in the case of a spherically symmetric index of refraction.

This problem can be reformulated in more general geometric terms as to whether given a compact Riemannian manifold with boundary one can determine the Riemannian metric in the interior knowing the lengths of geodesics joining points on the boundary. This is a problem that also appears naturally in rigidity questions in Riemannian geometry. It is known as the boundary rigidity problem.

In this talk we will discuss recent progress about the boundary rigidity problem.

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SERGIO VESSELLA

Università degli Studi di Firenze, Italy

Optimal three cylinder inequality for solutions to parabolic equations and stability estimates in inverse problems

In some recent papers ([1], [2]) on inverse elliptic and parabolic boundary value problems with unknown boundaries, some interesting relationship between sharp stability estimates and strong unique continuation properties of solutions to PDE have been point out and exploited. Following this line, in this talk, we present some optimal three cylinder inequalities for solutions to parabolic equations with time dependent coefficients and discuss some application of it to prove stability estimates to the problems of determination of unknown boundaries.

[1] Alessandrini G., Beretta E., Rosset E., Vessella S., Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) Vol. XXIX (2000) pp. 755-806.

[2] Canuto B., Rosset E., Vessella S., Trans. Am. Math. Soc. 354, n. 2, (2002) pp. 491-535.

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Michael Vogelius

Rutgers, The State University of New Jersey, USA

Blow-up of solutions to non-linear elliptic Neumann problems

In this talk I shall discuss some recent results concerning blow-up of solutions to boundary value problems of the form

$$\triangle u = 0 \quad \text{in } \Omega \ , \qquad \frac{\partial u}{\partial n} = \lambda f(u) \quad \text{on } \partial\Omega \ .$$

A particular case, namely that of $f(t) = e^{\alpha t} - e^{-(1-\alpha)t}$, $\lambda \to 0$, frequently shows up in the context of corrosion modelling . This exponential f is associated with a phenomenon of blow-up at certain points. It is hoped that detailed information about the blow-up behaviour will facilitate the imaging (detection) of highly corroding, inaccessible parts of the boundary.