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Parabolic integrodifferential identification problems related to memory kernels with special symmetries

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The aim of this paper is to generalize our previous results obtained in [1] to the case of the generalized corona $\Omega_\mu = \{x = (x_1, x_2, x_3) \in \mathbb{R}^3 : c\mu(|x|^{-1}x) < |x| < \mu(|x|^{-1}x)\}$, where $0 < c < 1$, $|x| = (x_1^2 + x_2^2 + x_3^2)^{1/2}$ and $\mu \in C^2(\mathcal{O})$, $\mathcal{O}$ being an open set containing $\partial B(0,1)$. We note that the two disjoint surfaces $|x| = c\mu(|x|^{-1}x)$ and $|x| = \mu(|x|^{-1}x)$, which the boundary $\partial\Omega_\mu$ consists of, are related by a dilatation.
We are interesting in identifying the unknown memory kernel $k(t,\rho)$, appearing in the following integro-differential equation of parabolic type

$$D_t u(t,x) = \mathcal{A}u(t,x) + \int_0^t k\Big(t-s, \frac{|x|}{\mu(|x|^{-1}x)}\Big)\mathcal{B}u(s,x)\,\mathrm{d}s + \int_0^t D_\rho k\Big(t-s, \frac{|x|}{\mu(|x|^{-1}x)}\Big)\mathcal{C}u(s,x)\,\mathrm{d}s + f(t,x), \quad \forall\,(t,x) \in [0,T]\times\Omega_\mu. \tag{1}$$

In equation (1) $\mathcal{A}$ and $\mathcal{B}$ are two second-order linear differential operators, $\mathcal{A}$ being uniformly elliptic and in divergence form, while $\mathcal{C}$ is a first-order differential operator.
Though our identification problem seems to be a simple generalization to the case of a generalized corona Ω_μ of the one dealt with in [1] related to a *spherical* corona Ω_1 the situation is much more complex.
We will single out a special class of *admissible* operators $\mathcal{A}$ and two pieces of suitable additional information for which the problem of identifying k can be uniquely solved locally in time at least when the domain Ω_μ is a *small* deformation of the spherical corona Ω_1.

[1] Favaron A., Lorenzi A.: *Parabolic integrodifferential equations related to radial memory kernels I,* J. Inverse Ill Posed Problems, 9 (2001), 489–529.